

Theoretical Review of Radiative Rare B Decays¹

Giulia Ricciardi

Dipartimento di Scienze Fisiche, Università degli Studi di Napoli,
and I.N.F.N., Sezione di Napoli,
Mostra d' Oltremare Pad. 19, I-80125 Napoli, Italy
e-mail address: gricciardi@axpna1.na.infn.it

Abstract

A status report on the theory of radiative rare B decays in the standard model is presented, with focus on inclusive decays $B \rightarrow X_{(s,d)} \gamma$ and exclusive decays $B \rightarrow (K^*, \omega, \rho) \gamma$. CP asymmetries are also briefly discussed.

1 Introduction

The radiative rare B decays have received a lot of attention in the recent years. Since they are forbidden at tree level in the standard model, they can occur only as loop effects and therefore give information on masses and couplings of virtual particles running in the loops, like the W or the t quark. Radiative rare B decays test QCD corrections and provide a searching ground for non standard physics and CP violating asymmetries.

Here a status report on the theory of radiative rare B decays in the standard model is presented. The effects of the strong interactions on the weak radiative B decays are studied in the framework of the effective hamiltonian. We discuss separately the inclusive decays $B \rightarrow X_{(s,d)} \gamma$ and the exclusive decays $B \rightarrow (K^*, \omega, \rho) \gamma$, and then briefly comment on CP violation asymmetries.

¹Invited talk at European Physical Society Conference on High Energy Physics, Brussels, July 1995; to be published in the proceedings.

2 Inclusive Decays $B \rightarrow X_{(s,d)}\gamma$

2.1 Rate

The inclusive decay $B \rightarrow X_s\gamma$ is described at the partonic level by the weak decay $b \rightarrow s\gamma$, corrected for short-distance QCD effects. Support for the spectator model in inclusive decays comes from the $1/m_b$ expansion[1]. The perturbative QCD corrections are important in this decay, enhancing the rate by 2–3 times, which makes the theoretical prediction compatible with the experimental rates within the errors.

The perturbative QCD corrections introduce large logarithms of the form $\alpha_s^n(\mu) \log^m(\mu/M)$ ($m \leq n$), where α_s is the strong coupling, M is a large scale ($M = m_t$ or M_W) and μ is the renormalization scale. By using renormalization group equations, the large logarithms are resummed order by order and the coefficients of the effective hamiltonian can be calculated at the relevant scale for B decays $\mu \sim O(m_b)$ [2]. Although the first analyses date back to many years ago, the first fully correct calculation at leading order (LO) of the anomalous dimension matrix has been obtained only in 1993[3] and confirmed last year[4]. The main problem has been the evaluation of the two loop diagrams, that mix the operators $(Q_1\dots Q_6)$ with the operators (Q_7, Q_8) (for the definition of $Q_1\dots Q_8$ see e.g.[3]). The effect of these diagrams has been found too large to be ignored. It should not be surprising that two loop diagrams are already present at the LO in QCD corrections, since this weak decay starts at one loop at order α_s^0 .

The next-to-leading order (NLO) calculation has been only partially completed. The two-loop mixing in the sector $(Q_1\dots Q_6)$ has been calculated[5], as well as the two-loop mixing in the sector (Q_7, Q_8) [6]. Gluon corrections to the matrix elements of magnetic penguin operators have also been calculated[7, 8, 9]. The $O(\alpha_s)$ corrections to $C_7(M_W)$ and $C_8(M_W)$ have been considered in ref.[10]. The three loop diagrams that mix the operators $(Q_1\dots Q_6)$ with the operators (Q_7, Q_8) are still to be calculated; as seen at LO, their contribution may be relevant and estimates based on the incomplete NLO calculation must be handled with care. At LO[11], it has been estimated

$$Br(B \rightarrow X_s\gamma)_{TH} = (2.8 \pm 0.8) \times 10^{-4}, \quad (1)$$

assuming $|V_{ts}|/|V_{cb}| = 1$, as suggested by the unitarity of the Cabibbo Kobayashi Maskawa matrix (CKM). After the inclusion of $O(\alpha_s)$ virtual and bremsstrahlung corrections and taking into account the scale dependence of the running quark masses, the prediction is[12]

$$Br(B \rightarrow X_s\gamma)_{TH} = (2.55 \pm 1.28) \times 10^{-4}. \quad (2)$$

The error is dominated by the uncertainty in the choice of the renormalization scale[9, 11, 12]. An attempt to apply scale fixing methods to this decay has been

done[13]. However, the last word will belong to the complete NLO calculation, that should considerably reduce theoretical uncertainties at LO. Within the large errors, the prediction is in agreement with the inclusive CLEO data[14]

$$\text{Br}(B \rightarrow X_s \gamma)_{EXP} = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}. \quad (3)$$

By factorizing the CKM parameters, the amplitude can be written as

$$A = v_u A_u + v_c A_c + v_t A_t, \quad (4)$$

where $v_u = V_{us}^* V_{ub}$, $v_c = V_{cs}^* V_{cb}$ and $v_t = V_{ts}^* V_{tb}$. Since v_u is negligible with respect to v_c and v_t ($|v_u| \sim O(\lambda^5)$ and $|v_c| \sim |v_t| \sim O(\lambda^2)$, in the parameterization of Wolfenstein), it follows that $v_c \sim -v_t$ by the unitarity of the CKM matrix. The amplitude is thus proportional to v_t and one can use this decay normalized to the semileptonic decay to estimate $|V_{ts}|/|V_{cb}|$ [12]:

$$|V_{ts}|/|V_{cb}| = 1.10 \pm 0.43. \quad (5)$$

Similarly, one could use $b \rightarrow d\gamma$ to extract information on $|V_{td}|$. This decay has not been detected. The expected branching ratio is approximately $O(10^{-5})$ [15]. Even though statistically the inclusive decay is in the reach of future CLEO/B factories, it is difficult to be observed, due to the large background of $b \rightarrow s\gamma$. In $b \rightarrow d\gamma$, the CKM factors in the effective hamiltonian have comparable size $|v_u| \sim |v_c| \sim |v_t| \sim O(\lambda^3)$; the proportionality to $|V_{td}|$ could thus be jeopardized by contributions coming from the c and u loops. At LO, however, there are no large contributions of the type $\alpha_s \log(m_u^2/m_c^2)$ [16].

Recently, the possibility of non perturbative long distance effects on the rate through resonant intermediate states has been suggested. These effects have been estimated by using the vector meson dominance hypothesis (VMD) or non relativistic quark models[16, 17, 18]. In the estimates by VMD, the radiative transition $b \rightarrow (s, d)\gamma$ is modelled by a sum over the processes $b \rightarrow (s, d)V_i^*$, where V_i^* is a virtual vector meson (ψ and its excited states, ρ , and ω), followed by the conversion $V_i^* \rightarrow \gamma$. In this picture, it is assumed that only the transverse part of $b \rightarrow s\gamma$ couples to the photon, in order to preserve gauge invariance. A crucial problem is that the effective couplings, that are measured at the mass of the resonance in the intermediate process, must be scaled to $q^2 = 0$, since the final photon is on shell. This scaling may lead to a strong suppression that reflects in small long distance contributions, less than 10% with respect to the short distance rate[16, 17]. The size of this suppression awaits further work; if absent, the rate becomes much bigger[18]. Similar results follow in the $b \rightarrow d\gamma$ case[16, 17].

2.2 Photon energy spectrum

In the two body decay $b \rightarrow s\gamma$ the photon energy is fixed. A non-trivial photon energy spectrum is obtained by

- including perturbative emission of hard gluons, such as $b \rightarrow s\gamma g$ [7, 9, 12, 15]
- taking into account the non perturbative motion of the b quark inside the meson [12, 19, 20].

The amplitude for the decay $b \rightarrow s\gamma g$ suffers from singularities in the limit of soft gluons or photons ($E_\gamma \rightarrow E_\gamma^{max}$ or $E_\gamma \rightarrow 0$, respectively). These singularities are cancelled in the photon energy spectrum if one also takes into account the virtual corrections to $b \rightarrow s\gamma$ and $b \rightarrow g\gamma$, order by order in perturbation theory. Near the endpoint regions, the spectrum can be improved by resumming at all orders the leading (infrared) logarithms [12, 20]. The region $E_\gamma \rightarrow E_\gamma^{max}$ deserves particular attention, since it is the region that contributes mostly to the rate. Note that in the limit $m_s \rightarrow 0$ also collinear singularities come into play [21].

In order to implement the B -meson bound state effects on the photon energy spectrum, one can use a specific wave function model [22], where the B -meson consists of a bound state of a b quark and a spectator quark of mass m_q . The b quark is given a momentum having a Gaussian distribution, centered around zero, whose width is determined by a parameter p_F . Both parameters of the model, p_F and m_q , can be fitted by the CLEO photon energy spectrum [12].

Another approach to the B -meson bound state effects is based on QCD and $1/m_b$ expansion [19, 20]. The spectrum is expressed in terms of a universal distribution function whose moments are related to local quark operators of increasing dimensions. This function depends on the final quark mass (e.g., it is different for $B \rightarrow X_s\gamma$ and $B \rightarrow X_c e \nu$). A few free parameters have to be determined by matching with the experimental data.

In inclusive $b \rightarrow (s, d)\gamma$ decays, the two approaches are compatible through the leading order in $1/m_b$ [23].

3 Exclusive Decays $B \rightarrow V\gamma$

The matrix element of the effective hamiltonian gives the so called short distance (SD) contribution to the amplitude for the exclusive decays. For the $B \rightarrow V\gamma$ decay (where $V = K^*, \rho, \omega$), the SD amplitude A is proportional to the matrix element of O_7

$$\begin{aligned}
\langle V(\eta, k) | \bar{s} \sigma^{\mu\nu} \frac{1 + \gamma_5}{2} q_\nu b | B(p) \rangle = & 2 T_1^{B \rightarrow V}(q^2) \epsilon^{\mu\alpha\rho\sigma} \eta_\alpha(k) p_\rho k_\sigma + \\
& i T_2^{B \rightarrow V}(q^2) [\eta^\mu(k) (m_B^2 - m_V^2) - (\eta(k) \cdot q)(p + k)^\mu] + \\
& i T_3^{B \rightarrow V}(q^2) (\eta(k) \cdot q) \left[q^\mu - \frac{q^2}{m_B^2 - m_V^2} (p + k)^\mu \right], \tag{6}
\end{aligned}$$

where $T_1^{B \rightarrow V}(q^2)$, $T_2^{B \rightarrow V}(q^2)$ and $T_3^{B \rightarrow V}(q^2)$ are real form factors, η is the V polarization vector and $q = p - k$ is the photon momentum. One can show that when the photon is on-shell $T_1^{B \rightarrow V}(0) = T_2^{B \rightarrow V}(0)$ and $T_3^{B \rightarrow V}$ does not contribute to A ; the rate depends thus on one form factor only.

Among exclusive decays, the $B \rightarrow K^* \gamma$ has received an increasing attention and the relative form factor has been calculated in a large number of papers (see e.g.[24] and references therein), employing several models: HQEFT, quark models, QCD sum rules, lattice. When comparing experiment and prediction, it is convenient to use the ratio $R_{K^*} = \Gamma(B \rightarrow K^* \gamma) / \Gamma(b \rightarrow s \gamma)$, that is largely independent from many theoretical uncertainties, like renormalization scale, unknown perturbative higher order corrections, etc. Experimentally, $R_{K^*} = (19 \pm 9)\%$ [25, 14]. Recently, QCD sum rule models have given results in the ball park of the experimental data[26, 27], e.g. $R_{K^*} = (16 \pm 5)\%$ [27] by using the light cone sum rules.

In lattice QCD, the point of physical interest is not directly accessible at present and one has to make an ansatz about extrapolating the results to $q^2 \rightarrow 0$ and to the physical b quark mass. In particular, the functional dependence of $T_{1,2}$ on q^2 is critical for the results, since a pole-like behaviour for $T_2^{B \rightarrow K^*}(q^2)$ results in $R_{K^*} \sim (4 - 13)\%$, while a constant behaviour for $T_2^{B \rightarrow K^*}(q^2)$ results in an appreciably higher rate. There are four groups working on this matrix element: BHS[28], UKQCD[29], LANL[30] and APE[31]; we limit here to report their results for R_{K^*} in the cases of $T_2^{B \rightarrow K^*}(q^2)$ pole dominance (first number) and $T_2^{B \rightarrow K^*}(q^2)$ constant (second number):

$$\text{BHS :} \quad (6.0 \pm 1.2 \pm 3.4)\%, \quad (7)$$

$$\text{UKQCD :} \quad (13 + 14 - 10)\% \quad (35 + 4 - 2)\%, \quad (8)$$

$$\text{LANL :} \quad (4 - 5)\% \quad (27 \pm 3)\%, \quad (9)$$

$$\text{APE :} \quad (5 \pm 2)\% \quad (31 \pm 12)\%. \quad (10)$$

The exclusive decay $B \rightarrow \rho \gamma$ has not been seen yet, but it is likely to be seen at future CLEO/B factories. It can be compared to $B \rightarrow K^* \gamma$ to extract information on $|V_{td}|/|V_{ts}|$. Although the form factors of $B \rightarrow K^* \gamma$ and $B \rightarrow \rho \gamma$ decays are model dependent, their ratio should be more reliable, being determined by $SU(3)$ symmetry considerations. Therefore, in the limit of SD contributions only and assuming that the top quark loop dominates the ratio, $|V_{td}|/|V_{ts}|$ can be estimated by

$$\frac{\Gamma(B_{(u,d)} \rightarrow \rho \gamma)}{\Gamma(B_{(u,d)} \rightarrow K^* \gamma)} = \frac{|V_{td}|^2}{|V_{ts}|^2} \xi \Omega, \quad (11)$$

where ξ is the squared ratio of the form factors and Ω is a phase space factor. The estimate is[32]

$$\frac{|V_{td}|}{|V_{ts}|} \leq (0.64 - 0.76), \quad (12)$$

where the range reflects the model dependence.

Until now we have assumed that only the matrix element of the magnetic moment operator O_7 contributes to the rate, neglecting the possibility of other long distance effects. In particular, beyond the spectator model approximation, the diagrams where the b quark annihilates the spectator quark by weak interaction have also to be considered. Because of the CKM matrix elements, this annihilation mechanism is negligible for $B \rightarrow K^* \gamma$, but it may be important for other exclusive radiative decays like $B \rightarrow \rho \gamma$. The presence of these effects may invalidate the relations (11) and (12).

Due to color suppression, the contributions of weak annihilation diagrams in neutral B decays $B^0 \rightarrow (\rho^0, \omega) \gamma$ are generally believed to be smaller than in the B^\pm case. Non spectator diagrams for $B^\pm \rightarrow \rho^\pm \gamma$ have been evaluated in a constituent quark model and found to change the decay rate by a factor of $0.7 - 2.5$ [33] with respect to the SD contribution. Estimates based on QCD sum rules find that the contribution of the weak annihilation diagrams modifies the SD rate of $B_u^\pm \rightarrow \rho^\pm + \gamma$ up to $\pm 20\%$ [34, 35].

The analysis of long distance contributions by VMD in exclusive decays $B \rightarrow (K^*, \rho) \gamma$ presents many theoretical uncertainties, like e.g. the role played by the spectator quark. These uncertainties reflect in a wide range of results for the VMD amplitude; it has been estimated to be from 5% to 50% of the SD amplitude[18, 36, 37].

We stress that in exclusive decays particular care must be exercised to avoid possible double counting among long distance effects.

4 CP Violation

CP violation in B radiative penguins decays may occur as interference among loop diagrams involving the u , c or t virtual quarks. Gluon exchange provides the necessary strong phase shifts between these diagrams. By using the unitarity of CKM matrix ($V_{ts}^* V_{tb} = -V_{us}^* V_{ub} - V_{cs}^* V_{cb}$), we can write the amplitude in the form

$$A = v_u A_u + v_c A_c, \quad (13)$$

where $v_u = V_{us}^* V_{ub}$ and $v_c = V_{cs}^* V_{cb}$. The CP violating asymmetry is

$$a_{CP} = \frac{\Gamma(\bar{B} \rightarrow \bar{f}) - \Gamma(B \rightarrow f)}{\Gamma(\bar{B} \rightarrow \bar{f}) + \Gamma(B \rightarrow f)}, \quad (14)$$

where $\bar{B} \rightarrow \bar{f}$ and $B \rightarrow f$ are CP conjugate processes. The asymmetry (14) can be written as

$$a_{CP} = \frac{-4 \operatorname{Im}(v_u v_c^*) \operatorname{Im}(A_u A_c^*)}{|v_u A_u + v_c A_c|^2 + |v_u^* A_u + v_c^* A_c|^2}. \quad (15)$$

The amplitude A_i has a strong phase that does not change sign in the transformation $A_i \rightarrow \bar{A}_i$, in contrast to the weak phase due to the Cabibbo Kobayashi Maskawa factors. At SD, the amplitude A is the matrix element of the effective hamiltonian between the initial and final state. At LO, only O_7 contributes to the decay, the amplitude is real and the asymmetry is zero; one needs to go at least at order $O(\alpha_s)$ in the matrix elements to create the phase difference between A_u and A_c . This asymmetry has been estimated[38] to be of the order $(0.1 - 1)\%$ for $b \rightarrow s\gamma$ and $(1 - 10)\%$ for $b \rightarrow d\gamma$. For exclusive modes, a_{CP} is typically 1% for $B \rightarrow K^*\gamma$ and 15% for $B \rightarrow \rho\gamma$ [39]. It is evident that the observation of a large asymmetry in $b \rightarrow s\gamma$ mediated processes would provide by itself strong evidence of physics beyond the SM.

The asymmetry (14) has been estimated in a constituent quark model, including the contributions due to non spectator diagrams, for $B^\pm \rightarrow \rho^\pm\gamma$; it has been found that it can be sizable, possibly as large as 30%[33].

The prospects for CP violation in B decay modes that are dominated by at least two interfering resonances have also been investigated[40]. It has been estimated that, in decays like $B \rightarrow K\pi\gamma$, $B \rightarrow K^*\pi\gamma$, $B \rightarrow K\rho\gamma$ and $B^\pm \rightarrow \pi^\pm\pi^\pm\pi^\pm\gamma$, CP violating distributions may be observed in a sample of about $10^8 - 10^9$ B^\pm mesons[40].

Acknowledgment

The author gratefully acknowledges useful discussions and e-mail exchanges with A. Ali, T. Bhattacharay, I. Bigi, F. Buccella, R. Gupta, A. Khodzhamirian, Z. Ligeti, A. Soni, A. Vainshtein, H. Yamamoto.

References

- [1] I. Bigi et al., in *Proceedings of the DPF meeting of APS*, Singapore, 1993, Eds. World Scientific, p. 610. A. Falk, M. Luke, M.J. Savage, *Phys. Rev.* **D49** (1994) 3367.
- [2] S. Bertolini et al., *Phys. Rev. Lett.* **59** (1987) 180. N.G. Deshpande et al., *Phys. Rev. Lett.* **59** (1987) 183. B. Grinstein, R. Springer and M.B. Wise, *Nucl. Phys.* **B339** (1990) 269. R. Grigjanis et al., *Phys. Lett.* **B213** (1988) 355. *Erratum* **B286** (1992) 413. G. Cella, G. Curci, G. Ricciardi, A. Vicer , *Phys. Lett.* **B248** (1990) 181.
- [3] M. Ciuchini et al., *Phys. Lett.* **B316** (1993) 127. M. Ciuchini et al., *Nucl. Phys.* **B421** (1994) 41.

- [4] G. Cella, G. Curci, G. Ricciardi and A. Viceré, *Phys. Lett.* **B325** (1994) 227. *Nucl. Phys.* **B431** (1994) 417. M. Misiak, *Nucl. Phys.* **B393** (1993) 23. *Erratum* **B439** (1995) 461.
- [5] A.J. Buras et al., *Nucl. Phys.* **B408** (1993) 209. M. Ciuchini et. al. *Nucl. Phys.* **B415** (1994) 403.
- [6] M. Misiak, M. Münz, *Phys. Lett.* **B344** (1995) 308.
- [7] A. Ali, C. Greub, *Zeit. Phys.* **C49** (1991) 431.
- [8] A. Ali, C. Greub, *Phys. Lett.* **B259** (1991) 182.
- [9] A. Ali, C. Greub, *Zeit. Phys.* **C60** (1993) 433.
- [10] K. Adel and Y.P. Yao, *Mod. Phys. Lett.* **8** (1993) 1679. *Phys. Rev.* **49** (1994) 4945.
- [11] A.J. Buras et. al., *Nucl. Phys.* **B424** (1994) 374.
- [12] A. Ali, C. Greub, preprint SLAC-PUB-95-6940 (1995).
- [13] J. Chay, S.J. Rey, preprint SNUTP 95-050 (1995).
- [14] M.S. Alam et al. (CLEO Coll.), *Phys. Rev. Lett.* **74** (1995) 2885.
- [15] A. Ali, C. Greub, *Phys. Lett.* **B287** (1992) 191.
- [16] G. Ricciardi, *Phys. Lett.* **B355** (1995) 313.
- [17] N.G. Deshpande, X. He, J. Trampetic, preprint OITS-564-REV (1995) (revised version).
- [18] D. Atwood, B. Blok, A. Soni, preprint SLAC-PUB-6635 (1994). J. M. Soares, preprint TRI-PP-95-6 (1995) (revised version).
- [19] I. Bigi et al., *Int. J. Mod. Phys.* **A9** (1994) 2467. M. Neubert, *Phys. Rev.* **D49** (1994) 4623.
- [20] R.D. Dikeman, M. Shifman, N.G. Uraltsev, preprint TPI-MINN-95-9-T (1995).
- [21] A. Kapustin, Z. Ligeti, H.D. Politzer, *Phys. Lett.* **B357** (1995) 653.
- [22] A. Ali, E. Pietarinen, *Nucl. Phys.* **B154** (1979) 512. G. Altarelli et al., *Nucl. Phys.* **B208** (1982) 365.
- [23] I. Bigi et. al., *Phys. Lett.* **B328** (1994) 431. G. Baillie, *Phys. Lett.* **B324** (1994) 446. C. Csaki, L. Randall, *Phys. Lett.* **B324** (1994) 451.

- [24] S. Veseli, M.G. Olsson, preprint MADPH-95-900 (1995). R.N. Faustov, V.O. Galkin, preprint JINR-E2-94-438 (1994).
- [25] R. Ammar et al. (CLEO Coll.), *Phys. Rev. Lett.* **71** (1993) 674.
- [26] P. Colangelo et al., *Phys. Lett.* **B317** (1993) 183. P. Ball, preprint TUM-T31-43-93 (1993). S. Narison, *Phys. Lett.* **B327** (1994) 354.
- [27] A. Ali et al., *Zeit. Phys.* **C63** (1994) 437.
- [28] C. Bernard, P. Hsieh, A. Soni, *Phys. Rev. Lett.* **72** (1994) 1402.
- [29] D.R. Burford et. al. (UKQCD), *Nucl. Phys.* **B447** (1995) 425.
- [30] T. Bhattacharay, R. Gupta (LANL), in *Lattice '95*, Melbourne (Australia).
- [31] A. Abada et al. (APE), preprint CERN-TH-95-59 (1995).
- [32] M. Athanas (CLEO Coll.), preprint CLEO-CONF-94-2(1994).
- [33] G. Eilam et al., preprint TECHNION-PH-95-4 (1995).
- [34] A. Ali, V. M. Braun, preprint DESY 95-106 (1995).
- [35] A. Khodzhimirian, G. Stoll, D. Wyler, *Phys. Lett.* **B358** (1995) 129.
- [36] N. G. Deshpande, J. Trampetic, K. Panose, *Phys. Lett.* **B214** (1988) 467. E. Golowich, S. Pakvasa, *Phys. Rev.* **D51** (1995) 1215.
- [37] H.Y. Cheng, preprint IP-ASTP-23-94 (1994); J. Milana, preprint UMD-PP-95-110 (1995).
- [38] J. M. Soares, *Nucl. Phys.* **B367** (1991) 575.
- [39] C Greub et al., *Nucl. Phys.* **B434** (1995) 39.
- [40] D. Atwood, A. Soni, *Zeit. Phys.* **C64** (1994) 241. *Phys. Rev. Lett.* **74** (1995) 220.